Reliable Broadcasting without Collision Detection*

Extended abstract

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Abstract. We propose a dynamic, ad-hoc communication network consisting of mobile units that can warn about traffic jams on motorways. Our goal is to provide a practical, low cost solution. Therefore we consider very simple wireless communication hardware, without collision detection, with very small bandwidth and a probabilistic model of link failure.
We provide a complete system architecture. For this purpose we design and analyze solutions for size approximation, leader election and broadcasting. Our algorithms are fine-tuned for fast operation in a practical setting. We provide both a theoretical and experimental evaluation of our solutions.
Our contribution is much different from the previous work, where either pure theoretical models with a pure theoretical analysis are provided or algorithms working in practical models are evaluated only through simulations.

1 Introduction

Communication in ad-hoc networks has been a broadly studied topic recently. A wide range of algorithms for routing ([6]), broadcasting ([8],[7]), MAC protocols ([2]) and algorithms for leader election ([5], [3]) and size approximation ([4]) have been proposed. Usually, the authors consider networks, where the number of participating nodes is a parameter n that can take arbitrary values. So the focus is on solutions that achieve good performance in parameters that depend on n.

A typical approach in the literature is to use layered solutions, where routing and broadcasting algorithms are layered on top of MAC protocols, which themselves use leader election algorithms for proper operation. This ensures ease of design and maintenance as each layer is responsible for assuring only a few properties. However, it degrades performance to some extent. This is a minor issue

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for modern communication hardware (e.g. WLAN) operating with high bandwidths. However, this becomes a significant problem, if we work with extremely simple hardware and a low-bandwidth communication channel. Moreover, we have to design algorithms having good performance for small parameter values, asymptotical behavior is less important.

Problem Statement. Our goal is to design an ad-hoc communication system for cars traveling along a motorway so that they can be warned about conditions ahead (like jams, accidents, ...). Certainly, there are hundreds of ways, in which one can deploy such a warning system. However our goal is to find a low-cost solution with a minimal number of resources used, and working in an environment with many faults. (Of course, the system will be designed in a much more general setting, we start with a concrete scenario in order to justify some parameter choices.)

We assume that no infrastructure can be used for the system – it consists solely of mobile nodes installed on mobile cars. The proposed system relies on very limited communication hardware with the following properties:

- transmission range is r,
- only one frequency channel is available for transmission,
- the available bandwidth is practically in the order of a few Kbits per second,
- the transmitters are synchronized,
- a transmission can be properly received, if it has been sent by a node within distance r from the receiver and no other transmitters are active during this time within the interference range of $2r$, i.e. no collision has occurred.
- collision detection is limited: only a sender of a message can check whether his message has been sent successfully, other nodes cannot distinguish between a collision, random noise and lack of transmission,
- transmissions are unreliable – i.e. a collision free transmission is received by a node with a constant probability p_r (this models imperfections of the wireless channel).

We design a complete system fulfilling the above goals and provide both theoretical and experimental analysis. The core of the system are algorithms for size approximation, leader election and broadcasting. These algorithms cooperate tightly to ensure that messages propagate fast along the motorway. In detail, we present:

- the overall system architecture for the interaction of all necessary algorithms (Section 2),
- a size approximation algorithm which runs in linear time with respect to the maximum number of stations in a sector (Section 3),
- a leader election algorithm which elects a leader in logarithmic time in terms of the maximum number of stations in a sector (Section 4),
- a reliable broadcasting algorithm (Section 5).

Section 6 concludes the paper with the results of an experimental evaluation.

Due to space limitations a proofs of several lemmas have been ommitted in this version of the paper and can be found in the full version.

2 System Architecture

The main functionality of the system is provided by a broadcasting algorithm (BA) which passes information from a traffic jam to other stations on the road. For the broadcasting algorithm, the road has been conceptually divided into geographical sectors of relatively small length, which is the half of the transmission range. This implies that cars must be able to determine the sector they are currently in (e.g. by utilizing GPS).

For the communication to work properly, we assume that all transmitters are synchronized. As there is only one frequency available for communication, a time-division protocol is used in order to split available transmission time.

It may not be allowed for all sectors to broadcast their message at the same time since the interferences would cause an excess of collisions. Thus each sector is allowed to transmit only in specific time slots. These time slots are allocated in such a way, that only sectors in a sufficient distance transmit in parallel. Thus each BCAST slot (as shown in Fig. 1) is divided into a constant number of sub-slots in which appropriate sectors can transmit. The same applies for size approximation and leader election.

Periodically, the number of nodes in each sector is estimated by the size approximation algorithm (SA). This estimation is used by the leader election algorithm (LEA). Before each time slot used for broadcasting there is a time slot devoted to leader election, so that during broadcasting there is a leader in each in each sector. Figure 1 illustrates this division.

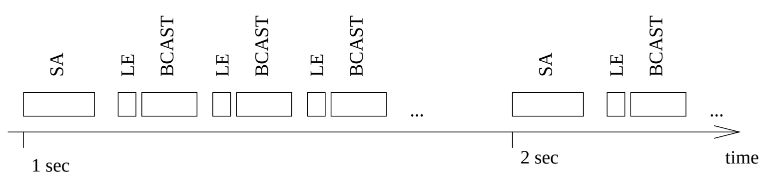


Fig. 1. Static time division between algorithms

The only algorithm utilizing communication between sectors is the broadcast algorithm. Size approximation and leader election work locally within respective sectors. This implies that these algorithms work in a one-hop wireless network.

3 Size Approximation

In this section we present an algorithm estimating the number of stations in a single-hop radio network. We assume that an upper bound on number of stations in a sector is known to each station (the number of cars in a sector is limited – this follows from physical sizes!). We do not demand from the nodes to have unique IDs.

In contrast to many previous solutions we do not pay attention to energy cost of our algorithms (i.e. the number of time slots, when a station transmits or listens to the communication channel), since in our case we have a large amount of energy available from the car's electrical system. Our aim is to get algorithms which are fast and reliable. We are interested in achieving good results for a small number of stations, without considering asymptotic complexity.

Multi-Round Algorithm. We call a transmission proper (or successful), if it does not collide with any other transmission. We divide stations in two groups: active stations and inactive stations. Inactive stations are those which are allowed to listen only. Active stations listen, too, but they can also transmit messages. (Simultaneous transmitting and listening is feasible in our case, since transmitter and receiver can be installed on different parts of a car.)

In our algorithm, the goal is that every station has exactly one successful transmission during the algorithm. Then we take the number of successful transmissions as an estimated number of stations within the sector. We will show that with high probability the number of successful transmissions is equal to the number of stations in the sector, if parameters are suitably chosen. In the following paragraphs we assume that the maximum number of stations in a sector is equal to N_{max}, the actual number of stations is N and, as a result of the algorithm each station knows n, an approximation of N.

The algorithm runs for a duration of f rounds, each consisting of M_j single-bit transmission slots. At the beginning, all stations are active. All stations (both active and inactive) listen all the time.

Before a round j, each active station i chooses uniformly and independently at random a transmission slot $T_{i,j} \in 1, \ldots, M_j$. Then, within round j, every active station i sends a single bit message in transmission slot $T_{i,j}$. When there are no collisions with other transmissions, i.e. $T_{i,j} \neq T_{k,j}$ for every $i \neq k$, the message is received properly by every station.

Every station counts the number of proper transmissions in each round. A station which has succeeded (i.e. its transmission was successful) becomes inactive. Active stations, which did not manage to successfully transmit in round j, remain active.

After f rounds, every station computes an estimation of the number of stations as the number of successful transmissions from all rounds.

Analysis of One Round. Now we shall investigate the properties of one round of the Multi-Round Algorithm. Based on that knowledge, we can set an optimal number and length of rounds.

Expected Number of Successful Transmissions. Let S^j denote the number of successful transmissions in the jth round.

Lemma 1. *Let N_j denote the number of active stations in the jth round. Let M_j be the number of transmission slots of the jth round. Then the expected number of new inactive stations after the j-th round is $N_j(1 - 1/M_j)^{N_j - 1}$.* □

Optimal Length of Rounds. Now we investigate an optimal trade-off between the number of transmission slots for a given round and the expected number of successful transmissions. For a fixed number of stations we want to maximize the expected number of successes per transmission slot for each round of the algorithm.

Consider the function $\mu(m,n) = n(1-\frac{1}{m})^{n-1}$ equal to the expected number of successful transmissions in one round, assuming there are exactly n active stations in the given round and m time slots reserved for that round. Defining a price function $p(m,n) = \frac{\mu(m,n)}{m}$ describing the average number of successes per step and finding its minimum we obtain the result $m = n$. Thanks to this, we have an indication on how to set the number of transmission slots per round to achieve a good time efficiency.

To obtain a sequence of round lenghts we set $M_1 = N_{max}$, and for the next rounds, we set the number of time slots equal to the expected number of active stations $M_{i+1} = M_i - \mu(M_i, N_i) = M_i - \mu(M_i, M_i)$.

Deviation from Expectation. The previous analysis bases only on expectations and is thus insufficient. In Lemma 2 we limit the deviation of S^j from its expectation. Its proof can be found in the full version of the paper.

Lemma 2.

$$\Pr[E[S^j]^2 - S^j \geq \lambda] \leq \exp\left(-\frac{\lambda^2 N^2 (2M-1)}{2(M-1)^2(N^2 - E[S^j]^2)}\right) . \tag{1}$$

By Lemma 2 we estimate how many stations will become inactive after one round with high probability. We define $\lambda(m,n,\delta)$ to be a number λ such that Eq. 1 is satisfied for $M = m$, $N = n$ and probability not greater than δ. With this in mind, we can redefine the price function $p(m,n) = \frac{\mu(m,n)-\lambda(m,n,\delta)}{m}$. Choosing for each round such a length M_j that $p(M_{j-1}, N_{j-1})$ is maximized and setting $N_j = N_{j-1} - (\mu(M_j, N_{j-1}) - \lambda(M_j, N_{j-1}, \delta))$ we obtain a sequence of round lengths $\{M_1, \ldots, M_k\}$ which allow the algorithm to count the number of nodes precisely with probability greater than $1 - k\delta$. Unfortunately the bound on the deviation as presented in Lemma 2 is not applicable for $N \leq 10$ as it is not tight enough to produce reasonable results.

Link Unreliability. In the proofs we have omitted the problem of link unreliability to avoid complex analysis. Because every node hears proper transmission with probability p_r, the expected estimated value is $\geq p_r N$.

Evaluation. Here we give some calculated examples of the time efficiency of the proposed algorithms. We assume a maximum of 100 stations in one sector and try to minimize the time needed for the size approximation in such a setting. The analysis based on expectations gives us a sequence of round lengths $\{100, 63, 40, 25, 16, 10, 6, 4\}$ with a total length of 264 time slots. Obviously the analysis taking deviations into account suggests us to allocate more time slots, in detail this is $\{140, 106, 90, 75, 78, 55, 48\}$ with a total length of 592 slots.

We have performed 100000 independent experiments for the worst case of $n = 100$ and the second sequence of round's lengths. In all cases the precise number of nodes was computed by the algorithm. In the same setting, the first sequence achieves a probability of 0.45 to estimate the number of stations perfectly.

4 Leader Election

The algorithm consists of one-bit transmission slots, called steps. During one step each node transmits messages with probability $1/n$, where n denotes the node's estimation of the total number of nodes in the sector. The first node, which successfully transmits its message becomes the leader.

During the remaining steps every node that knows that a leader has been elected sends messages in each step in order to prevent the other stations from becoming a leader.

For the analysis of the leader election algorithm see the full version of this paper.

5 Broadcast

The broadcast algorithm works as follows. The key role of the stations in each sector is to forward messages to the next sector. The only active station in each sector in a given point of time is the leader of this sector. Thus it is his responsibility to resend a received message. Leaders should resend a message so that a sector as a whole resends the message a predefined constant number of times.

Each node remembers the messages it has seen coming from the previous sector (those which should be forwarded). It also remembers all messages it has heard from the leader of its section, which it has not heard from the previous sector. It remembers also for each message how many times it was heard from a leader of its sector. If a message is resent often enough (a constant number of times), the node marks it as processed. The message is also marked as processed when the station leaves the sector.

Analysis. Due to the properties of the communication model, the analysis of the broadcast algorithm is probabilistic. The reliability of the algorithm is measured in terms of the probability of successful transmission of a message between two distant sectors in a given time.

The key problem which underlies the broadcast reliability is the link unreliability. After a transmission from sector S only a fraction of all stations in sector $S + 1$ knows about the transmitted message and can transmit it to the next sector.

In order to simplify the analysis we make two assumptions. First, we disregard node mobility (in terms of nodes leaving and joining a sector). We will show that a message is with high probability transmitted to the next sector within two time slots. Even with low bandwidth for transmissions these two time slots correspond to a fraction of a second – only a small number of stations can change sectors in this time. The experimental evaluation presented in Section 6 takes node mobility fully into account. Second, we assume that only one message is traveling through the system.

We are going to analyze what happens when a message reaches some sector S, i.e. it is transmitted by sector $S-1$. The number of stations which have heard this transmission is described by the binomial distribution with success probability p_r (recall that p_r denotes the probability of hearing a successful transmission) and n trials. Then the probability that exactly k of n stations received a message correctly is equal to $\binom{n}{k}p_r^k(1-p_r)^{n-k}$. The probability that this message is retransmitted by sector S in the next broadcast slot is equal to $\frac{k}{n}$, as we assume that the leader election algorithm chooses uniformly at random one of the stations and k of n stations know about the message.

So the probability that a message is passed from the Sth sector to the $S+1$st sector, after there has been a transmission from the sector $S-1$, is equal to:

$$\sum_{k=0}^{n}\binom{n}{k}p_r^k(1-p_r)^{n-k}\frac{k}{n}=\frac{1}{n}(1-p_r)^n\sum_{k=0}^{n}\binom{n}{k}k\left(\frac{p_r}{1-p_r}\right)^k=p_r\ .$$

Let us describe this value by $P_1^{p_r,n}$. With $P_k^{p_r,n}$ we describe the probability that a message is passed successfully from sector S to sector $S+1$ for the first time in the kth broadcast slot ($k-1$ previous steps were unsuccessful).

The problem with values $P_k^{p_r,n}$ is that they are hard to compute. Thus in Lemma 4 we develop an approximation for them, based on auxiliary Lemma 3.

Lemma 3. *For every i,n and every series k_i it holds*

$$\left(1-\sum_{i=1}^{l}\frac{k_i}{n}\prod_{j=1}^{i-1}\left(1-\frac{k_j}{n}\right)\right)=\prod_{j=1}^{l}\left(1-\frac{k_j}{n}\right)\ .$$

□

Lemma 4. *For all l and n,*

$$\sum_{i=1}^{l}P_i^{p_r,n}\geq\sum_{k=0}^{n}\sum_{i=1}^{l}\binom{n}{k}p_r^k(1-p_r)^{n-k}\frac{k}{n}(1-\frac{k}{n})^{i-1}\ .$$

□

Lemma 5. *For all $p_r\in(0,1)$ there exists a constant c such that for all $n\geq 1$ and $l\geq 1$ the following inequality holds:*

$$\sum_{k=0}^{n}\sum_{i=1}^{l}\binom{n}{k}p_r^k\left(1-p_r\right)^{n-k}\frac{k}{n}\left(1-\frac{k}{n}\right)^{i-1}\geq c\sum_{i=1}^{l}p_r(1-p_r)^{i-1}\ .$$

Proof. The inequality has been numerically evaluated for all $l \geq 1$ and $n \geq 1$. c is an increasing function of n and l for all $n \geq 1$ and $l \geq 2$. For $l = 1$ the sums are equal. Thus c can be chosen as the value required for $l = 2$ for the smallest n used practically. □

Now, we are ready to investigate the reliability of the algorithm. We will consider random variables $F(i, j)$, where $F(i, j)$ is the probability that a message travels from the sector S to sector $S + i$ in exactly j broadcast slots. It is easy to see that the following recursive equation holds

$$F(i,j) = \begin{cases} \sum_{k=i-1}^{j-1} F(i-1,k) P_{j-k}^{p_r,n} , & \text{if } i > 1 \\ P_j^{p_r,n} , & \text{if } i = 1 \ . \end{cases}$$

Of course, $F(i, j) = 0$ for $j < i$.

We are interested in finding the value of $\sum_{t=0}^{m} F(i, i + t)$, which defines the probability that a message travels the requested distance in $i + m$ or less transmission slots. Unfortunately, the values $F(i, j)$ are hard to compute (mainly because of $P_{j-k}^{p_r,n}$). Thus we will substitute $F(i, j)$ with $T(i, j)$. We define

$$T(i,j) = \begin{cases} \sum_{k=i-1}^{j-1} T(i-1,k) p_r (1-p_r)^{j-k-1} , & \text{if } i > 1 \\ (1-p_r)^{j-1} p_r , & \text{if } i = 1 \ . \end{cases}$$

In Lemma 6, we show that the series $T(i, j)$ is a lower bound for $F(i, j)$.

Lemma 6. *For given p_r, and c from Lemma 5, for all i, m*

$$\sum_{j=1}^{m} F(i,j) \geq c^i \sum_{j=1}^{m} T(i,j) \ .$$

□

By Lemma 6 we can use $T(i, j)$ instead of $F(i, j)$.

It is easy to see that $T(i, j) = w(i, j)p_r^i (1 - p_r)^{j-i}$ as there should be exactly i successful transmissions and exactly $j - i$ failures. One can derive values $T(i, j)$ from $T(i - 1, k)$, for $i - 1 \leq k \leq j - 1$. The following equality holds for $i \geq 2$:

$$T(i,j) = \sum_{k=i-1}^{j-1} T(i-1,k)(1-p_r)^{j-k-1} p_r \ .$$

Thanks to that, one can find the coefficient $w(i, j)$:

$$\begin{aligned} T(i,j+1) &= \sum_{k=i-1}^{j} T(i-1,k)(1-p_r)^{j-k} p_r \\ &= (1-p_r) \sum_{k=i-1}^{j-1} T(i-1,k)(1-p_r)^{j-k} p_r + p_r T(i-1,j) \\ &= (1-p_r) T(i,j) + p_r T(i-1,j). \end{aligned}$$

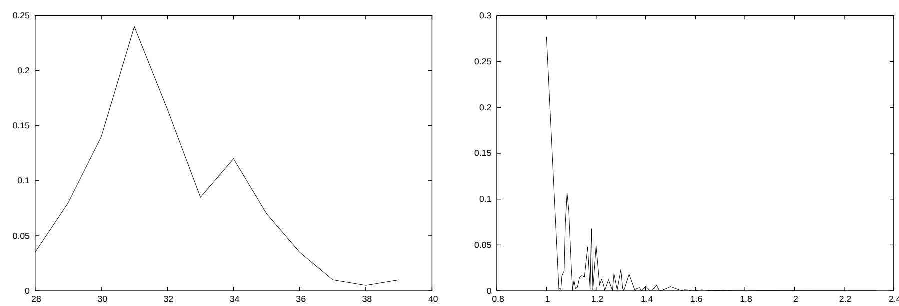
Now, while comparing coefficients w, we get $w(i,j+1) = w(i,j) + w(i-1,j)$, hence $w(i,j) = \binom{j-1}{i-1}$. Finally, we get

$$T(i,j) = \binom{j-1}{i-1} p_r^i (1-p_r)^{j-i} \ .$$

Thus we can lower bound the value of

$$\sum_{t=0}^{x} P(i,i+t) \geq c^i \sum_{t=0}^{x} T(i,i+t) = c^i \sum_{t=0}^{x} \binom{i+t-1}{i-1} p_r^i (1-p_r)^t \ .$$

Evaluation. With the assumption of $p_r = 0.9$ the value of c from Lemma 5 has to be set to 0.9818. This gives a probability of traveling a distance of 10 sectors within 20 slots of time equal to approximately 0.83. For practical parameters (sector length 250 meters and 10 broadcast time slots per second) this distance and time correspond to 2.5 kilometers and 2 seconds. If we additionally assume that there are at least 10 cars in each sector (for crowded traffic very common) then $c = 0.9909$ and thus the probability of traveling 10 sectors in 20 time slots increases to 0.91.



(a) Distribution of travel time of broadcast messages

(b) Distribution of approximation ratio of size approximation

Fig. 2. Reliability of algorithms in simulation

6 Experimental Evaluation

The system has been evaluated experimentally within an environment based on a cellular automaton of the Helbing type (see [1]), which simulates the behavior of cars on a road.

The cellular automaton has been extended by allowing each car to run its own simulated wireless transceiver. Additionally each car is able to run the size approximation, leader election and broadcasting algorithms.

The goal of the simulator was to measure message travel time, leader election success rate and size approximation accuracy. The test environment consists of a road of 6.75 km length, thus being divided into 27 sectors. Every 3 seconds a broadcast is issued by the last sector and is forwarded by the broadcasting algorithm to the first sector. Figure 2(b) shows the distribution of the travel time of messages from the last sector to the first one. As there are 10 broadcast time slots in each second, the most common travel time of 31 time slots corresponds to 3.1 seconds. The p_r parameter has been set to 0.9.

The cars on the road travel with a maximum speed of 135 km/h, with a density of 0.3 and all other cellular automaton parameters as suggested in [1].

The leader election success rate is equal to 0.99959 and the size approximation accuracy is shown in Fig. 2(a). For every invocation the leader election algorithm was allowed to run for 20 rounds and the size approximation algorithm was allowed to run for 264 rounds, divided into 8 rounds as suggested in Section 3.

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