Exercise 1.

In this exercise you will implement some basic linear algebra using C++. In particular, you will implement a few functions that perform some useful operations on mathematical vectors. We will use `vector<double>` in order to represent a mathematical vector $v \in \mathbb{R}^n$. Write a program that implements a function for each of the following tasks. Check your function implementations by calling them on small test data as shown in the following code snippet:

```cpp
#include <cmath>
#include <iostream>
#include <vector>
using namespace std;

void print_dvector(const vector<double> &v) {
    for (const double &d : v) {
        cout << d << ' ';
    }
    cout << '
';
}

double euclidean_length(const vector<double> &v);
double scalar_product(const vector<double> &v, const vector<double> &w);
```


```cpp
#include <iostream>
#include <vector>

using namespace std;

vector<double> normalize(vector<double> v);

double euclidean_distance(const vector<double> &v, const vector<double> &w);

int main()
{
    vector<double> a = {1, 2, 3};
    vector<double> b = {4, 5, 6};
    // You have to provide the implementations for the four function declarations
    // in above to make this code work.
    cout << "length of 'a': " << euclidean_length(a) << "\n";
    cout << "scalar product of 'a' and 'b': " << scalar_product(a, b) << "\n";
    print_dvector(normalize(a));
    cout << "distance between 'a' and 'b': " << euclidean_distance(a, b) << "\n";
    return 0;
}
```

Implement a function called ...

a) **euclidean_length** that computes the euclidean length of a vector. (1 P.)

The euclidean length of a vector \( \mathbf{v} \in \mathbb{R}^n \) is defined as
\[
||\mathbf{v}|| = \sqrt{\sum_{i=1}^{n} v_i^2}.
\]

b) **scalar_product** that computes the scalar product of two vectors. (1 P.)

The scalar product \( \langle \cdot, \cdot \rangle \) of two vectors \( \mathbf{x}, \mathbf{y} \in \mathbb{R}^n \) is defined as
\[
\langle \mathbf{x}, \mathbf{y} \rangle = \sum_{i=1}^{n} x_i \cdot y_i.
\]

c) **normalize** that computes a normalized version of a vector. (1 P.)

A normalized vector can be obtained by dividing each of its entries by its (euclidean) length.

d) **euclidean_distance** that computes the euclidean distance of two vectors. (1 P.)

The euclidean distance of two vectors \( \mathbf{x}, \mathbf{y} \in \mathbb{R}^n \) is defined as
\[
||\mathbf{x} - \mathbf{y}||_2 = \sqrt{\sum_{i=1}^{n}(x_i - y_i)^2}
\]

**Exercise 2.**

Fibonacci numbers are numbers from an integer sequence, called Fibonacci sequence. Every number in this sequence is the sum of the two preceding ones:
\[1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, \ldots\]

The Fibonacci sequence \( F_n \) can be defined by the following recurrence relation (recursion):
\[F_1 = 1, \ F_2 = 1, \ F_n = F_{n-1} + F_{n-2}\]

a) Implement the function `unsigned fibonacci_rec(unsigned n)` such that it calculates the \( n \)-th Fibonacci number by using the recursive definition from above. (2 P.)

b) Implement the function `unsigned fibonacci_nonrec(unsigned n)` such that it computes the \( n \)-th Fibonacci number but uses sequential code rather than recursion. (Hint: use three variables and a loop inside the function body) (2 P.)

c) Compute the 50-th Fibonacci number using both of your Fibonacci implementations. Is there a noticeable difference in the runtime? Why does the recursive version take so much longer? (Note, it is not the negligible overhead caused by a function call.) (2 P.)
Exercise 3.
Declarer a variable `mymap` of type `std::map<string, int>` which is declared in the standard template library.
(Use `#include <map>`.) Please refer to http://en.cppreference.com/w/cpp/container/map for how to use `std::map`. You can find detailed descriptions as well as example code. You should have a look at the member functions (constructor), `operator[]` and the corresponding examples.

a) Add the following tuples to `mymap` that map a person's name to their age: ("Peter", 40), ("Brian", 4), ("Stewie", 1), ("Chris", 15), ("Meg", 14). (1 P.)

b) Write a function that prints the contents of `mymap` to the command line. (2 P.)

c) Add the tuple ("Lois", 41) to `mymap` and print the contents of the map again. (1 P.)

Exercise 4.
In the lecture we have already learned that pointer and reference types can be quite useful. We also discussed that one can represent points-to relationships as a graph. Consider the following (not very useful) code snippet:

```cpp
int i, j, k;
int *a = &i;
int *b = &k;
int **p = &a;
int **q = &b;
int *c = *q;
```

a) Watch the following video that provides an excellent introduction to pointers: https://youtu.be/Rxvv9krECNw?t=4m18s (0 P.)

b) Draw the corresponding directed graph that captures the points-to relations of the above code snippet. Use nodes to represent variables and directed edges to represent points-to information. Write the variables name and its type in each node. (2 P.)